Mast scantlings,
the not so noble, but practical art of intelligent guessing.

From day one of my fiddling with junk rigs, I’ve always felt on thin ice when trying to decide the scantlings for my unstayed masts. For the first solid spruce mast I designed for Malena (1990), I simply used the graphs in Hasler/McLeod’s Practical Junk Rig, PJR. Although this mast held up well, it soon became obvious that it was on the heavy side. When mast no. 2 (hollow spruce) was made for her in 1995, I pinched her diameter a little, and when Johanna’s mast was designed in 2000, I cut off even more on diameter. Still it was mostly based on guesswork.

The problem I found with the graphs in the PJR is that the boats righting moment, $M_R$, is not counted with, only the mast’s length above partners, LAP, and sail area, SA. Obviously the authors assumed that the readers would choose some sensible SA/displacement at say 13 to 17. Such common sense was not for me — I gave the 1500kg Malena a sail area of 32sqm. This means that her SA/Displ. = 24!

In the following I will try to describe a method for calculating the scantlings of an unstayed wooden mast where the boat’s $M_R$ and wood strength, $\delta$ (sigma) are the main deciding factors. SA is not important. Whether you have to reef in F3 or F5 doesn’t matter; the load on the mast at that point will be the same.

The strength of a round pole:
The maximum breaking strength for a hollow round pole is given by the formula:

$$M_B = \delta \cdot \left( \frac{\pi}{32} \cdot \frac{D^4 - d^4}{D} \right)$$

..where ...

- $\delta$ [kp/m\(^2\)] is breaking stress of the material; here: wood (tear, break or compression).
- $D$ [m] is outer diameter of the pole.
- $d$ [m] is inner diameter of the pole, i.e. D minus two times the wall thickness.
- $(1kp=1$ kilopond= 1 kilogram force.)

The solid pole is just a special case of the hollow pole where the inner diameter, $d$, is reduced to 0.

The reference pole of 20cm diameter:
The formula above is quite awkward to work with. To save you time, I have calculated the breaking moment for a 20cm solid and a hollow pole made from moderately strong spruce.

From Ian Nicholsons book, Boat Data Book, I pick the lowest stress value for spruce; the crushing strength $\delta = 457\, kp/cm^2 = 4,570,000\, kp/m^2$

Then the breaking moment of the hollow pole of 20cm will be (wall thickness is 20% of diameter):

$$M_{B/20cm,Hollow} = 4,570,000 \cdot \frac{\pi}{32} \cdot \frac{0.20^4 - 0.12^4}{0.20} = 3124\, kpm$$
and for the 20cm solid pole it will be:

\[ M_{B/20cm,\text{Solid}} = 4.570.000 \times \frac{\pi}{32} \times \frac{0.20^4 - 0.0^4}{0.20} = 3589\text{kpm} \]

From this reference point it is very easy to calculate the strength of any spruce mast. Just follow these two rules:

1. The breaking moment of a mast varies with the CUBE of the diameter.
2. The diameter of a mast varies with the CUBE ROOT of the breaking moment.

**Example:**
Let’s say you need a solid spruce mast with a breaking moment of 12000kpm. The diameter of the mast will then (..from rule no.2 above...) be:

\[ D_{12000\text{kpm/S}} = 20\text{cm} \times 3 \sqrt[3]{\frac{\text{Needed breaking moment}}{\text{Breaking moment at 20 cm}}} = 20\text{cm} \times 3 \sqrt[3]{\frac{12000}{3589}} = 29.9\text{cm} \]

**Another one:**
You have a hollow spruce mast of 27cm and wonder how strong it might be. Use rule no.1 above and find the breaking moment to be:

\[ M_{B/27cm,\text{Hollow}} = M_{B/20cm,\text{Hollow}} \times \left( \frac{\text{Your mast}}{20\text{cm}} \right)^3 = 3124 \times \left( \frac{27\text{cm}}{20\text{cm}} \right)^3 = 7686\text{kpm} \]

**The righting moment of a boat:**
The righting moment of a boat is:

\[ M_R = \text{Displacement} \times \text{righting lever} \]

That righting lever varies with the heel of the boat and is not easy to decide. Unless we have the actual static stability data of the boat, one will have to do some guessing. When seeing such data sheets it strikes me how short the righting lever is; generally well less than a quarter of the boats beam. In lack of more firm data I just set that maximum lever to just \(1/4\) of the boat’s beam, hence:

\[ M_R = \text{Displacement} \times 1/4 \times \text{Beam} \]

So now we are ready for the hard part:

**Finding a sensible safety factor:**
Obviously we cannot dimension the mast to break at the boat’s maximum righting moment. We need a “safety factor”, \( F_S \) so that:

\[ M_B \geq F_S \times M_R \]

The only way to find a sensible value for that “Safety Factor” (.. in quotation marks — it should rather be called a Fudge Factor or Experienced Factor...), is to use the numbers of known boats and calculate their \( F_S \):

**Case Malena:**
Disp=1500kg, Beam=2.24m, Mast=21cm hollow spruce.
This gives righting moment= 840kpm and...
the mast’s breaking moment=3616kpm, alas..

“Safety Factor”, \( F_{S/Malena} = \frac{3616\text{kpm}}{840\text{kpm}} = 4.30 \)
Using the same method for the 3.3ton/2.53m Johanna, the 5ton/2.44m Zuleika and 2.1ton/2.2m(?) Jester, their factors gets:
\[
\begin{align*}
F_s/Malena &= 4.30 \\
F_s/Johanna &= 2.92 \\
F_s/Zuleika &= 1.77 \\
F_s/Jester &= 2.01
\end{align*}
\]
This leads me to conclude:
- **Malena**’s 21cm mast is stronger than needed and adds quite a lot to the heeling moment. The PJR graphs still suggest a diameter of 23.2cm!
- **Johanna**’s \( F_s \) confirms my hunch that her 25cm mast is plenty strong enough. At 30deg. heel the mast robs 10 – 20% of the boat’s righting moment. Just acceptable for cruising. The PJR graphs suggest a diameter of 27.4cm.
- **Zuleika**’s 24cm hollow mast has proven that it stands up to offshore cruising. Don’t argue with success. The PJR graphs suggest 23.5cm.
- The two Jesters have lost 2 masts. The first was in a rollover, so it doesn’t count. The last mast (diam.=18.14cm) that broke in fair weather, was built up from staves. It seems that it broke due to an inferior scarf joint. The PJR graphs suggest 22.1cm.

It is now up to you to pick a “Safety Factor” for your next rigging project.

**Finally some practical stuff:**
The diagram shows how I build a dug-out spruce mast. It is a copy of the PJR practice. The diameter in mast top is 0.4 of the diameter at partners and at the step it is 0.5 (square).
If a dug out version is chosen, half-finish the mast as a roughly dug-out job while the pole is raw (.. I cheat and leave that job to a boatbuilder...). Then it dries faster and doesn’t develop shakes. If you glue it up from staves yourself, you MUST make a practice run on a test pole of about 2m first with one scarf joint in each stave. On that finished mast EACH AND EVERY scarf joint must be perfect!
I cover the mast with one layer of glass fibre roving in epoxy, and finally paint or varnish with 2-part polyurethane paint or varnish to stop the sunrays.

I have skipped one moment in the design process: the mast’s slenderness factor:
\[
F_{si} = \frac{LAP}{\text{Diameter}}
\]
An increase of the \( F_{si} \) obviously will lead to an increase of the stress created by the boat’s violent pitching.

Here are the \( F_{si} \)-values for...

- **Malena**: \( 8.4/0.21=40 \)
- **Johanna**: \( 9.4/0.25=38 \)
- **Zuleika**: \( 9.14/0.24=38 \)
- **Jester**: \( 9.4/0.18=52 \)
I can’t prove anything, but I guess, for offshore work I would consider keeping the \( F_{si} \) below 45. In that case the Jester mast would grow to a diameter of 20.9cm. That looks sensible to me. In other words, my advice is:
Make the mast as short as you can,
and then...
.. pile on as much sail as you can!

Stavanger, 11.12.2006
Arne Kverneland

PS:

**Collecting more data:**
Obviously the few examples I have shown here are not enough to build up a good reference base to aid calculating the scantlings of masts. I therefore hope that many of you will collect the basic data about your boat (boat type, length, beam, draft, displacement, ballast etc.) and mast(s), including steel and aluminium (material, outer D, wall thickness, LAP etc.) and e-mail these data to me. That would be helpful.

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